

# Design of Elastic Mode Suppression Systems for Ride Quality Improvement

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**An analytical technique is presented for design of systems to actively control the structural bending modes to improve the ride quality on flexible aircraft. The technique specified a set of requirements for the sensor configuration and filtering for a system which provides corrective control through the conventional aerodynamic control surfaces. Vehicle data are required only in a frequency response format, thus facilitating the inclusion of unsteady aerodynamics. Application of the technique to a large supersonic transport is described. Gust response data on the SST indicated that the lower frequency symmetric structural modes contributed up to 60% of the rms acceleration at the pilot's station. Results of the study showed that a system designed by the synthesis technique essentially achieved the performance objective of a 50% reduction in the rms acceleration level at the pilot's and other critical vehicle stations. This application of the synthesis technique to the SST problem showed it to be an effective design tool.**

## Introduction

IN large aircraft such as the supersonic transport (SST) the ride quality deteriorates in turbulence because of the increased vehicle flexibility. To alleviate this problem, the response amplitudes of the significant structural modes can be reduced through use of an active control system. The control system is designed to sense the structural motions and to generate a corrective control force through the aerodynamic control surfaces. To date, design of such a system, referred to as an elastic mode suppression system, has been accomplished primarily by trial-and-error techniques. The presence of factors such as unsteady aerodynamics and the relatively large number of parameters to be evaluated make these techniques cumbersome and limited in application. A synthesis technique has been developed which results in an orderly and systematic procedure for defining an elastic mode suppression system.

This paper describes the synthesis technique by first discussing the general mode suppression problem in terms of the design objectives and design approach considerations that form a basis for the development of an elastic mode suppression system. Using this information, the formulas and conditions for the synthesis procedure are established. Finally, the use of the synthesis technique is demonstrated through an example using the SST with four symmetric structural modes. Although the synthesis technique described herein was evolved from development work on a large supersonic transport, it is intended for a more general application.

Application of the synthesis technique to the SST illustrated its effectiveness, not only in deriving a solution to the problem, but in other aspects. The performance results demonstrated that a significant degree of ride improvement can be attained through the use of an active control system. For the SST application, a design goal of a 50% reduction in the rms acceleration levels in turbulence was essentially achieved at the critical vehicle stations.

## The General Problem

Ride quality is measured in terms of the rms acceleration level occurring at various crew and/or passenger stations in the vehicle. Stations at which the acceleration levels are large in terms of human comfort ratings are termed "sensitive" stations. The acceleration experienced at these stations in air turbulence is a result of the transfer of energy from the surrounding atmosphere to the vehicle, not only through the rigid body motion, but also through excitation of the vehicle's structural modes. The ride quality can be improved significantly over that of the unaugmented vehicle by using a rigid-body gust alleviation system and/or an elastic mode suppression system. Only the design of an elastic mode suppression system is considered in this discussion. It is not the objective herein to develop a specific elastic mode suppression system (EMSS) but rather to describe a synthesis technique that can be used to define such a system. This discussion will establish the design objectives and design approach upon which the EMSS design is based. Hence, these assumptions serve also as a basis for development of the synthesis technique.

Gust response data on vehicles such as the SST and XB-70 indicate that the lower-frequency symmetric structural modes contribute significantly to the acceleration levels at the sensitive fuselage stations (e.g., the pilot's station and the aft passenger cabin). For example, in one of the configurations of the SST it was estimated that about 60% of the acceleration level experienced at a sensitive station could be attributed to the first three structural bending modes, about 30% to the rigid body motion, and the remaining 10% to the higher-frequency structural modes. Thus, it was evident that if the ride quality of the flexible vehicle was to be enhanced, an additional design objective was needed for the automatic flight control system. This design objective was to reduce the rms acceleration level at the sensitive fuselage stations by decreasing the response amplitudes of the significant structural modes, i.e., usually the lower-frequency modes. Uncertainties in the vehicle dynamics and increased system complexity make it unattractive to provide active mode suppression for the less significant higher-frequency modes in order to alleviate the additional contribution that they make to the acceleration levels. Thus it was an objective to minimize any coupling through the elastic mode suppression system with these higher modes.

Further, it was essential that satisfactory rigid-body dynamics be maintained while decreasing the response amplitudes of the significant structural modes. This is important

Received May 31, 1967; revision received September 29, 1967. The author wishes to acknowledge the vehicle data and technical assistance provided by the Lockheed-California Company for this paper. In particular, he would like to thank E. O. Thordson, J. A. Flapper, and H. P. Weinberger for their very significant contributions. He wishes also to express appreciation to J. C. Larson and R. C. Hendrick of Honeywell for their contributions through their leadership and technical assistance.

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from the standpoint of assuring good pilot handling qualities and to prevent amplified turbulence response at the short-period frequencies.

An additional design objective was to establish an analytical design which allowed an economical and reliable implementation. System complexity had to be minimized not only to enhance the reliability but to facilitate comparison monitoring of the system signals for failure detection. Compensating networks had to be defined which were realistic in terms of maintaining performance in the presence of tolerance effects accompanying the environmental conditions in which the aircraft must operate. The response amplitudes of the structural modes could be reduced by 1) a reduction in the amount of energy transferred to the structural modes from the input disturbance and 2) a rapid dissipation of the energy absorbed by the mode.

There are several disadvantages or limiting factors in using a system whose design is based wholly on one or the other of these two fundamental approaches. It was concluded that a system should be designed to make use of the features of both approaches. Considering the first approach, the only means for reducing the amount of energy being transferred is to apply a cancelling signal from some other source of energy, e.g., a control surface deflection. However, this approach relies heavily upon an accurate knowledge of vehicle dynamics. This is undesirable because of the uncertainty with which the vehicle dynamics are known. Then, too, a system based on cancellation would, more than likely, require complex scheduling of its parameters because of its sensitivity to variations in vehicle dynamics. Considering the second approach, dissipation of the energy, once it has been absorbed by the mode, can be achieved by augmenting the stability of the modes. Design of a system to provide stability augmentation tends to assure maximum tolerance to variations in vehicle dynamics, which is most desirable. But experience has shown that when two (or more) structural modes are close together in frequency it is difficult (because of coupling effects) to achieve a sufficient increase in the stability to reduce significantly the response amplitudes of the structural modes. A compromise between these two approaches can result in the simultaneous reduction of the structural mode response amplitudes at the sensitive fuselage stations and retention of adequate tolerance to uncertainties or changes in the vehicle dynamics.

To summarize at this point, it is evident that the following design objectives should be used to form the basis for the development of an effective elastic mode suppression system: 1) Reduce the response amplitudes of the significant structural modes (generally the lower frequency modes) in terms of the rms acceleration level at all sensitive fuselage stations where ride improvement is a requirement. 2) Provide maximum tolerance to unpredictable variations in vehicle-system dynamics. 3) Maintain adequate rigid-body dynamics in terms of good handling qualities and in terms of preventing amplified turbulence response at the short-period frequencies. 4) Minimize unfavorable coupling with the structural modes above the frequency range of the significant structural modes. 5) Establish an analytical design that permits an economical and reliable hardware implementation.

A necessary condition for suppression of structural flexure is the ability to sense the deflections, or a derivative of the deflections, of the significant modes and to apply control forces to these modes. The deflection rate and/or acceleration of the modes can be measured by conventional sensors (such as accelerometers) suitably located on the vehicle. A corrective control force can be applied through the deflection of one or more of the aerodynamic control surfaces. Hence, a design approach was assumed which would concentrate on the development of a system using conventional sensors to measure the modal deflection rates and/or accelerations and to command corrective action through the aerodynamic control surfaces.

The ride quality can be improved by the EMSS independent of the rigid-body stability augmentation system (herein referred to as the rigid-body SAS). The EMSS functions independently of the SAS to suppress the accelerations at the sensitive fuselage stations due to structural vibrations. To this end, both the rigid-body SAS and the autopilot are designed to provide absolute gain stability for all bending modes through the use of low-pass filtering. Conversely, the elastic mode suppression system is designed to have little or no effect over the rigid body frequencies. This approach, however, does not exclude the requirement for a coupled analysis of both the rigid-body SAS and the EMSS. There are several advantages to be realized by this separation of system functions:

1) This approach permits isolation of problem areas and facilitates preliminary selection of system parameters for both the rigid body SAS and the EMSS.

2) Greater flexibility is permitted in the design of both systems. Filter design, sensor placement, and sensor type need not be compromised between rigid-body requirements and mode suppression requirements. Sensors for the rigid-body SAS can be located to minimize the structural mode components in their output signals, whereas EMSS sensors can be combined to minimize pickup of rigid body motion. This approach provides a natural decoupling of the two systems. Further, control surfaces to be used in either system can be selected on the basis of their effectiveness for their particular function. Investigations have shown that the surfaces most suitable for control of the short-period mode are often not the most suitable for control of the structural modes and vice versa.

3) Impact of equipment failures is greatly reduced and efficient utilization of equipment redundancy according to the value of the function can be realized. For example, the rigid-body SAS is considered essential for good control, thus often justifying a triple redundant system for fail operational performance. The mode suppression system is not considered as essential, and thus usually requires only a dual redundant mechanism for failsafe operation.

The variations in vehicle mass, mass distribution, aerodynamics, and atmospheric turbulence require a highly tolerant system to provide satisfactory performance over the entire flight regime. The tolerance of the system depends on the nature of the design. However, it is evident that system complexity can be minimized if active augmentation is applied only over a limited portion of the flight regime. By avoiding an active control attempt over the entire flight regime, the adaptation problem is minimized. Thus, a real need for an active controller at a given flight condition must exist before the added complexity and associated costs can be justified.

In the preceding paragraphs the design objectives and design approach for the elastic mode suppression system were established. This information is applied in the following paragraphs to derive the formulas and constraints used in the synthesis technique.

### Development of the Synthesis Technique

In general, the synthesis technique used to design an EMSS should have the following properties: 1) facilitate design studies using a very high-order mathematical model of the vehicle, 2) enable use of this model in a frequency response form, thereby facilitating incorporation of nonsteady aerodynamic effects, 3) capability to compute required controller characteristics directly from specification of meaningful design criteria, and 4) permit rapid and accurate evaluation of non-ideal controller characteristics. The synthesis technique described on the following pages is an attempt to provide a design procedure with those properties.

The technique consists of 1) the specification of the control system filter, and 2) the specification of sensor configuration. The basic formula used in the synthesis technique specifies the

control system filter (i.e., compensation) requirements in terms of a given sensor configuration and an allocated attenuation requirement on the rms acceleration level at a sensitive fuselage station (e.g., the pilot's station). Additional requirements are specified in terms of stability constraints and in terms of restrictions resulting from the use of practical (in terms of mechanization) linear filters. These additional requirements enable the designer to specify a sensor configuration. As a consequence, once a reduction in the rms acceleration level (a measure of the ride quality) has been established at the sensitive fuselage stations, it is possible to specify a sensor-filter combination. The synthesis procedure requires a description of the vehicle dynamics in a frequency response format relating output variables of modal accelerations to input variables of wind gust and control surface deflection. Specification of data in this form facilitates incorporation of unsteady aerodynamics, a factor often not included in existing techniques.

For the sake of clarity, it is assumed in the discussion that the first three structural modes are the significant modes in terms of ride quality and, hence, are to be controlled by the EMSS. Actually, any number of modes may be used, but it should help the reader in following the development of the synthesis technique and in the subsequent example to talk in terms of a specific number of structural modes instead of, for example,  $n$  significant modes.

The approach to be taken toward performance specification for purposes of controller synthesis is to specify the desired reduction in acceleration at each of the resonant peaks of the first three modes. The frequencies of these peaks will be determined from a frequency response plot between normal accelerations and wind velocity, thereby including all known coupling effects. The required percentage reduction in amplitude at each frequency will be estimated from power spectral density plots of acceleration under turbulence at sensitive vehicle stations. Knowing the rms acceleration and identifying its source in terms of individual modes, a design goal for reduction of resonant amplitude can be allocated to each mode. An important part of this allocation is the definition of modes that do not cause significant acceleration contribution, so that the required control action at those particular frequencies can be relaxed. The objective of this process is to produce an appropriate allocation of control effort.

With this information, the controller can be specified (as will be shown) and then evaluated at other sensitive stations, and stability margins can be established. If all constraints are not met, the allocation of required reductions in amplitudes at the resonant modes can be modified and the process repeated until all the constraints are satisfied. This results in an iterative process, but with an orderly and systematic means of attack. The relating of peak accelerations to rms accelerations and the specification of reductions required in terms of acceleration at a sensitive station are useful methods of expediting the control synthesis process. The final measures of control effectiveness are the actual rms accelerations at the various vehicle stations.

To establish the basic formulas, consider a general block diagram of the control system-vehicle dynamics as shown in Fig. 1a. This diagram illustrates the relationship (in transfer function form) between the acceleration level measured at a sensitive fuselage station  $\ddot{Z}_c$  and the wind input  $W_g$ . Clearly, the diagram could be expanded to include all the sensitive fuselage stations, but this is not necessary for the following discussion. The term  $\ddot{Z}_s$  is the quantity to be measured by a sensor located at some vehicle station. The quantity  $K_c G(S)$  is the filter to be designed for the EMSS. Thus, it is the task of the synthesis process to determine what quantity will be sensed ( $\ddot{Z}_s$ ) and to define a filter  $K_c G(S)$  that will result in a specified reduction in the acceleration  $\ddot{Z}_c$  in response to the turbulence input  $W_g$ . It is assumed that the aircraft model is known in a frequency response form relating output variables of acceleration to input variables of wind turbulence  $W_g$  and

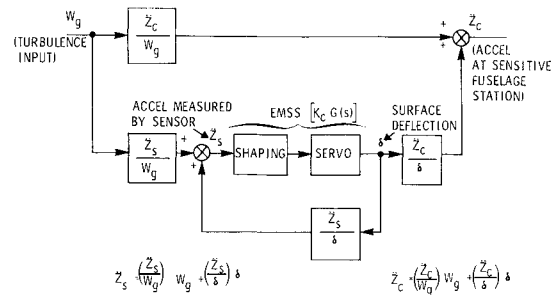


Fig. 1a General block diagram of vehicle-system model.

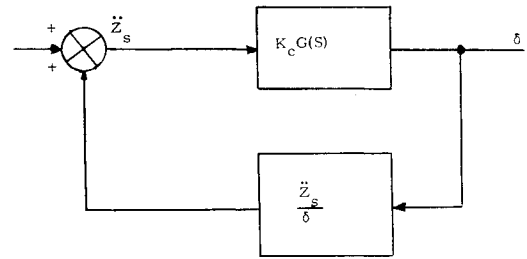


Fig. 1b Feedback loop.

control surface deflections  $\delta$ . Actuation system dynamics can either be included in the aircraft model or be left to be specified as part of the filter  $K_c G(S)$ . For the discussion it has been assumed that a satisfactory rigid-body SAS has been defined with appropriate filtering to attenuate adequately the stability augmentation signals at the structural mode frequencies.

With the design goals in terms of a reduction in acceleration at a sensitive station with augmentation to that without augmentation, one can derive an expression for the required filter characteristics  $K_c G(S)$ . Let  $Y$  equal the ratio of acceleration at a sensitive fuselage station with augmentation to that without augmentation. Now the acceleration occurring at the sensitive station  $\ddot{Z}_c$  without augmentation is simply

$$\ddot{Z}_c|_{\text{no aug}} = (\ddot{Z}_c/W_g)W_g \quad (1)$$

The acceleration at the sensitive station with augmentation is

$$\ddot{Z}_c|_{\text{with aug}} = \left\{ \frac{\ddot{Z}_c}{W_g} + \left( \frac{\ddot{Z}_s}{W_g} \right) \times \left[ \frac{K_c G(S)}{1 - (\ddot{Z}_s/\delta)K_c G(S)} \right] \left( \frac{\ddot{Z}_c}{\delta} \right) \right\} W_g \quad (2)$$

Thus, by the definition of  $Y$ , we obtain

$$Y = \left\{ \left( \frac{\ddot{Z}_s}{W_g} \right) \left[ \frac{K_c G(S)}{1 - (\ddot{Z}_s/\delta)K_c G(S)} \right] \left( \frac{\ddot{Z}_c}{\delta} \right) / \left( \frac{\ddot{Z}_c}{W_g} \right) + 1 \right\} \quad (3)$$

This definition of  $Y$  implies that  $Y$  can be a complex number. The designer is free to pick any  $Y$ , real or complex, within certain limits. Clearly, if the control is to reduce the amplitude of the acceleration at the discrete frequency points, then the absolute magnitude of  $Y$  chosen must be less than 1 (i.e.,  $0 \leq |Y| \leq 1$ ). Selecting real values for  $Y$  will result in lower gain values for the filter  $K_c G(S)$  than will the selection of complex values. Low gain values for the filter are desirable from the standpoint of minimizing the coupling with the rigid body mode and the high-frequency structural modes. We can solve Eq. (3) for  $K_c G(S)$  to yield

$$K_c G(S) = \frac{1}{\ddot{Z}_s} \left[ \frac{1}{1 - \frac{1}{1 - Y} \left( \frac{\ddot{Z}_c}{\delta} \right) \left( \frac{\ddot{Z}_s}{W_g} \right)} \right] \quad (4)$$

This equation defines the filter characteristics  $K_c G(S)$  in a frequency response form. It implies that if one picks the quantity to be measured  $\ddot{Z}_s$  and a value of  $Y$  at discrete frequency points (e.g.,  $Y_1, Y_2, Y_3$  corresponding to the first three structural mode resonant frequencies), then a gain and phase requirement can be computed for  $K_c G(S)$  at these discrete frequencies. If a filter can be designed to exhibit these gain and phase characteristics, then the acceleration  $\ddot{Z}_c$  will be attenuated by the values of  $Y$  chosen at these frequency points. It is sufficient to specify a filter requirement at only the resonant frequencies. Thus, the resulting filter will not resemble the inverse of the vehicle transfer function, as would probably be the case if the accelerations were to be reduced by only a cancellation technique.

The remaining task is to establish a procedure for defining the quantity to be measured  $\ddot{Z}_s$ . The objective is to specify the relative modal components of  $\ddot{Z}_s$  and not the type of sensor nor the combination of sensors required to actually provide  $\ddot{Z}_s$ . The sensor combinations can be established by examining the properties of  $\ddot{Z}_s$  and comparing them with the mode shapes.  $\ddot{Z}_s$  is considered to be an acceleration quantity have the following form:

$$\ddot{Z}_s = \ddot{Z}_0 + K_\theta \ddot{\theta} + \sum_{i=1}^n K_i \ddot{\eta}_i \quad (5)$$

where  $\ddot{Z}_0$  represents the rigid-body plunging mode,  $\ddot{\theta}$  represents the rigid-body rotation mode, and  $\ddot{\eta}_i$ 's represents the structural modes.

In the discussion that follows,  $\ddot{Z}_s$  is referred to as the sensor complement. Thus, to define  $\ddot{Z}_s$  corresponds to defining the relative values of the modal deflections,  $K_i$ . To do this we examine the following constraints:

1) A stability constraint is imposed on the system by the stability limits of the feedback loop shown in Fig. 1b (see Fig. 1a). This loop has the closed-loop transfer function given by

$$H(S) = \frac{K_c G(S)}{1 - (\ddot{Z}_s/\delta) K_c G(S)} \quad (6)$$

$H(S)$  is unstable whenever the roots of its denominator lie in the right-half plane of a root locus diagram. The denominator will be unstable at the frequency at which

$$\phi_{s\delta} + \phi_\sigma = 0^\circ \quad (7)$$

and

$$|(\ddot{Z}_s/\delta) K_c G(S)| \geq 1 \quad (8)$$

where  $\phi_{s\delta}$  = phase angle of the transfer function  $(\ddot{Z}_s/\delta)$  and  $\phi_\sigma$  = phase angle of the filter  $K_c G(S)$ .

Since one cannot reduce the amplitude of the bending mode accelerations by gain stabilization, it is necessary to provide phase stabilization by proper selection of  $\phi_{s\delta} + \phi_\sigma$ . Because of uncertainties in the mathematical model, it is necessary to require that, over the range of frequencies where gain stabilization is not attainable nor desirable, the following constraint be applied:

$$90^\circ \leq \phi_{s\delta} + \phi_\sigma \leq 270^\circ \quad (9)$$

This constraint assures that the stability of the modes will at least be increased and provide a minimum phase margin of  $90^\circ$ . At frequencies above the modes of interest (i.e., above frequencies where augmentation is to be applied) it is desirable to provide gain stabilization to eliminate this constraint on the phase angle.

With the aforementioned constraint [Eq. (9)] on the phase angle of the sensor-filter combination, the problem is to find a solution that, over the frequency range of interest, will satisfy simultaneously this constraint and the phase constraint imposed by the filter equation [Eq. (4)]. To impose the constraints defined by Eqs. (4) and (9), we observe that

$\phi_{s\delta} + \phi_\sigma$  = phase angle of

$$\left[ \frac{1}{1 - \frac{1}{1 - Y} \left( \frac{\ddot{Z}_c/\delta}{\ddot{Z}_c/W_\sigma} \right) \left( \frac{\ddot{Z}_s/W_\sigma}{\ddot{Z}_s/\delta} \right)} \right] \quad (10)$$

Defining

$$N = \frac{|\ddot{Z}_c/\delta|}{|\ddot{Z}_c/W_\sigma|} \quad V = \frac{|\ddot{Z}_s/W_\sigma|}{|\ddot{Z}_s/\delta|}$$

$$\phi_1 = [\text{phase of } (\ddot{Z}_c/\delta) - \text{phase of } (\ddot{Z}_c/W_\sigma)]$$

$$\phi_2 = [\text{phase of } (\ddot{Z}_s/W_\sigma) - \text{phase of } (\ddot{Z}_s/\delta)]$$

and restricting  $\phi_s + \phi_\sigma$  according to Eq. (9), we obtain a minimum attainable value on  $Y$ , the attenuation factor;

$$Y \geq 1 - VN \cos(\phi_1 + \phi_2) \quad (11)$$

At any flight condition,  $N$  and  $\phi_1$  are defined for a given vehicle station. Thus, if we pick a quantity to be measured,  $\ddot{Z}_s$ , we can determine the lower bound on  $Y$  (defined as  $Y_{\min}$ ) in order to assure that the stability constraint [Eqs. (4) and (9)] is satisfied. That is,

$$Y \geq Y_{\min} = 1 - VN \cos(\phi_1 + \phi_2) \quad (12)$$

If we assign a value to  $Y$ , say  $Y_i$ , at a given frequency ( $\omega_i$ ) then we must find a quantity to be measured  $\ddot{Z}_s$  such that

$$Y_i \geq Y_{\min}(\omega_i) \quad (13)$$

or else we cannot simultaneously satisfy the stability constraint and provide the attenuation  $Y = Y_i$ . The preceding condition is a necessary condition for specifying  $\ddot{Z}_s$ , but it is not a sufficient condition.

2) To specify further the requirements for the quantity to be measured,  $\ddot{Z}_s$ , it is necessary to examine other design objectives and analyze the practical constraints imposed on the filter  $K_c G(S)$ . The following design objective imposes a severe constraint on the type of filters and on  $\ddot{Z}_s$ : minimize unfavorable coupling with structural modes above the frequency range of the first three modes. To assure that this objective is met we impose the condition

$$|\ddot{Z}_s/\delta| |K_c G(S)| < 0.5$$

at modal frequencies above the third mode.

3) Now it is assumed the filter  $K_c G(S)$  will be a linear filter made up of first- or second-order elements and that the poles of the filter will be stable. Thus, the design objective of high-frequency gain stabilization, for practical purposes, imposes an additional constraint on  $K_c G(S)$ . To meet this objective, the gain of the filter will be decreasing with increasing frequency at a rate of at least 6 db per octave over the frequency range of the significant modes (e.g., the first three modes). From this, it becomes evident that if the same amount of attenuation (or more) is required at the second or third mode than as is required at the first mode frequency, then

$$|K_i| \geq (f_i/f_1) |K_1| \quad \text{for} \quad i = 2, 3$$

(where  $K_i$  is the  $i$ th mode deflection) to compensate for the loss in gain due to the filter  $K_c G(S)$ . For gain stabilization of the fourth and higher modes (assuming three significant modes) it is generally desirable to minimize  $K_i$  for  $i = 4, 5, \dots$

4) A further consideration is the amount of phase or gain change with frequency required of the filter  $K_c G(S)$ . Sensor configurations should, in general, be chosen to minimize large phase or gain change requirements with frequency for the filter  $K_c G(S)$  to facilitate the hardware implementation.

5) It is an objective to minimize the coupling with the rigid-body mode. This can be accomplished by placing the sensor in a position where the accelerations due to flexibility are the

largest with respect to those due to rigid-body motions. The coupling can be reduced further by minimizing the gain of the filter at the rigid-body frequencies.

All of the preceding considerations enable one essentially to define the desired sensor complement  $\check{Z}_s$ . To summarize, the sensor complement should be chosen to satisfy the following constraints: 1)  $Y_{\min} \leq Y_i$ ; 2)  $|K_i| \geq f_i/f_1|K_1|$ ,  $i = 2, 3$  (assuming 3 significant modes); 3) minimize the variation in gain and phase changes with frequency required of the filter  $K_c G(S)$ ; 4) maximize the ratio of acceleration due to flexibility (of the significant modes) over the acceleration due to rigid-body motions measured by the sensor(s); and 5) minimize the gain of the sensor-filter combination at frequencies beyond the significant structural modes.

Once the quantity to be measured,  $\check{Z}_s$ , has been defined, (according to the aforementioned rules) realistic sensor locations and combinations of sensors can be examined to find one that will best provide  $\check{Z}_s$ . With the actual sensor configuration defined, the filter requirements can be computed for that sensor configuration through the use of Eq. (4). Using these filter requirements, a practical filter can be designed. Finally, with the filter designed, the problem can be worked in reverse to check the "goodness" of the entire system. Thus, compromises in filter design can be analyzed to determine their effect on performance. It must be kept in mind that Eq. (4) does not have to be satisfied at all frequencies nor is its solution the only solution to the problem. Once a solution has been obtained, variations can be made on it to yield a possible better solution.

### Application to a Large SST Vehicle

The synthesis technique is demonstrated through an example using the SST vehicle configuration (Fig. 2) as the model vehicle. For the example, only the pitch axis was considered with the assumption of quasisteady aerodynamics. Four symmetric structural modes were represented, each with an assumed structural damping ratio of 0.015. The following paragraphs describe the design of an EMSS for the SST at the Mach 0.54, 5000-ft flight condition. A heavy configuration (585,000 lb) was selected. This represented one of the worst conditions in terms of turbulence effects. The system defined at this condition was also evaluated at the Mach 0.9, 30,000-ft flight condition to determine its tolerance.

Before the synthesis technique was applied, a qualitative analysis was made to determine the best control surface(s) to use and to establish any constraints on the sensor configuration which would be peculiar to the SST vehicle. There are four separate aerodynamic control surfaces distributed along the trailing edge of each wing (Fig. 2). The pair farthest outboard (pair 4) is the most attractive for use in the mode suppression system. Examination of the mode lines (ignoring aerodynamics) for the symmetric bending modes shows that pair 4 is the only pair that definitely qualifies for effective force application. A possible alternate for a somewhat restricted portion of the flight regime would be surface pair 3. Using pair 4 offered several advantages. These advantages include:

- 1) No direct conflict in authority requirements arises since the outer elevons (pair 4) were employed only during the gear-down operating condition. Pair 3 is dwelled between Mach 0.95 and 2.4.
- 2) Coupling with the rigid-body SAS is minimized because of the reduced surface effectiveness at the rigid-body frequencies.
- 3) Use of the outer surface pair has the advantage in that the phase relationship between the generalized forces and the aerodynamic control forces is more consistent (that is, the aerodynamic control forces tend to act only on the control surface itself). It is less affected by unsteady aerodynamic effects and by changes in flight condition because of the relatively short wing chord at the wing tip.

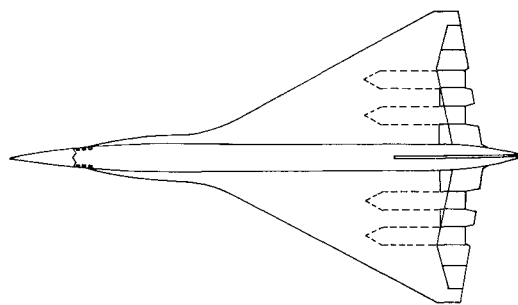


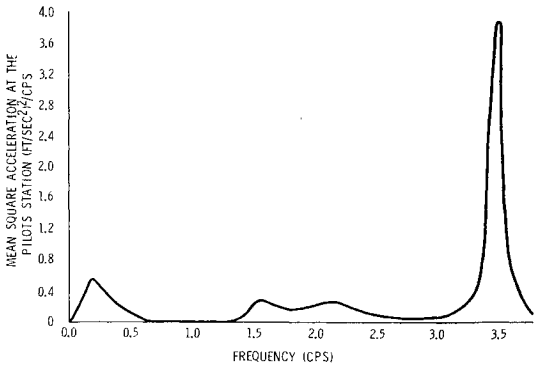
Fig. 2 SST vehicle configuration.

4) The outboard surfaces have the least inertia. This results in a lower ratio between inertia forces (due to control surface masses) and the generalized aerodynamic forces, thus the "tail-wag-dog" (TWD) effects are minimized. This TWD effect is shown in the system transfer function by a pair of lightly damped zeros. Because of their low damping ratio and their variation in frequency with surface effectiveness, electronic compensation is difficult. A TWD frequency in excess of the active mode suppression range appears essential if mode suppression is to be performed using aerodynamic surfaces. This frequency is kept high by minimizing the surface inertia.

5) Use of the surface pair 4 requires the addition of appropriate series servos to actuate these surfaces. This has the advantage in that there would be no conflict with servos presently being used for rigid-body stability augmentation. Two disadvantages are the requirements for a high-temperature servo and the added weight penalty. Use of the inboard surfaces may offer some benefit, particularly in providing some force decoupling among the bending modes; but for this example, only the surface pair 4 was considered since it appeared to be the most promising candidate for effective force application.

Linear accelerometers have a significant advantage over rate gyros for use as the bending mode sensors on the SST. Accelerometers have superior high-temperature capabilities, thus facilitating their placement near the outer elevons. Because of the temperature problem, placement of rate sensors would be restricted essentially to the fuselage. This restriction greatly reduces the potential capability of the rate sensor in the mode suppression system, except in a possible combination with an accelerometer. Examination of the mode shapes along the fuselage indicated there was no single rate sensor location that would provide proper phasing for all the first four structural modes. Two or more rate sensors could be blended, but examination of the mode shapes indicates that the resulting signal would be rather sensitive to variation in conditions. Placement of an accelerometer near the outboard surface pair (impractical for a conventional rate sensor because of temperature problems) tends to provide more uniform phasing between the control force applied and the resulting modal deflections. For these reasons, it was concluded that accelerometers offer the greater advantage as a candidate sensor.

The acceleration levels experienced by the free airplane in turbulence can readily be determined from power spectral density (PSD) plots of the mean square acceleration. These plots were obtained using the wind model described in Appendix A. Figure 3 is a PSD plot of the acceleration at the pilot's station at the Mach 0.54, 5000-ft flight condition. The frequency range of the plot covers the significant response amplitudes up through the fourth structural mode. The pilot's station and the aft cabin were found to be the two most sensitive fuselage stations in terms of a degradation in the ride quality due to flexibility on the SST. Calculation of the rms acceleration level from Fig. 3 shows it to be on the order of 0.04 g's/ft/sec rms of gust amplitude. Thus, for a



**Fig. 3 Mean square acceleration at the pilot's station vs frequency for a unit gust input.  $M = 0.54$ ;  $h = 5,000$  ft;  $W = 585,000$  lb.**

nominal gust amplitude of 4 fps, the rms acceleration occurring at the pilot's station would be 0.16  $g$ 's. According to information contained in Refs. 1 and 2, 0.1 $g$  is approximately the upper level for acceptable passenger or crew comfort in the low frequency range. Thus, it is desirable not to exceed this level. Although the duration of turbulence has not been taken into account, it appears that an rms acceleration level on the order of 0.16  $g$ 's is large in terms of passenger comfort. Sufficient data was not available to establish a specific design objective in terms of the amount of reduction required in acceleration levels at the sensitive stations over the flight envelope. However, the preceding order of magnitudes indicate that a design objective of a 50% reduction in the acceleration levels due to structural mode responses is a reasonable goal.

For the example, the following values of  $Y$  were assumed.

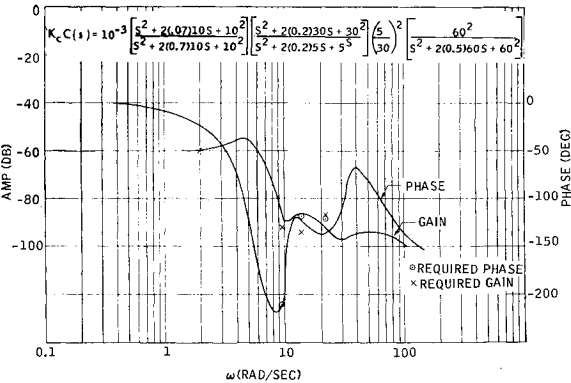
Bending frequencies	Attenuation factor
at $f = 1.55$ cps	$Y_1 = 0.5$
$f = 2.15$ cps	$Y_2 = 0.5$
$f = 3.5$ cps	$Y_3 = 0.25$

(14)

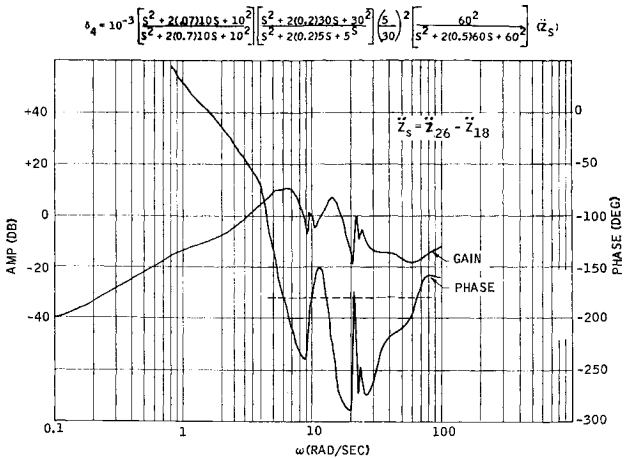
Application of the synthesis technique established the following set of requirements for the sensor complement: 1)  $\ddot{Z}_0$  term be minimized, 2)  $K_{\ddot{z}}$  be minimized, 3)  $K_2/K_1 \geq 1.4$ , 4)  $K_3 < 0$  and  $|K_3| \geq 2|K_1|$ , and 5)  $K_4 > 0$ . A sensor complement was selected which tends to satisfy these requirements. It was as follows:

$$\ddot{Z}_s = (\ddot{Z}_{26} - \ddot{Z}_{18}) = -477.2\ddot{\eta}_0 + 1.479\ddot{\eta}_1 + 3.34\ddot{\eta}_2 - 3.625\ddot{\eta}_3 + 0.651\ddot{\eta}_4 \quad (15)$$

This sensor complement was made up of three accelerometers. An accelerometer was placed at the two wing tips and their signals were averaged together to provide the net acceleration  $\ddot{Z}_{26}$ . A third accelerometer was placed near the aft



**Fig. 4 Frequency response of  $K_cG(S)$ .**



**Fig. 5 Open loop frequency response of vehicle-controller dynamics.**

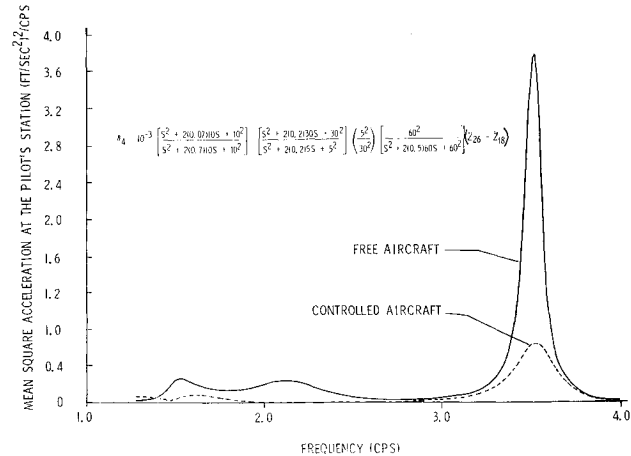
end of the fuselage ( $\ddot{Z}_{18}$ ). With this sensor complement and the values of  $Y$  given by (14) the following filter requirements were specified by Eq. (4). At

$$\begin{aligned} f = 1.55 \text{ cps} \quad & |K_cG(S)| = 0.252 \times 10^{-4} [\text{rad}/(\text{ft}/\text{sec}^2)] \\ & \phi_\theta = 214^\circ \\ f = 2.15 \text{ cps} \quad & |K_cG(S)| = 0.199 \times 10^{-4} [\text{rad}/(\text{ft}/\text{sec}^2)] \\ & \phi_\theta = 117^\circ \\ f = 3.5 \text{ cps} \quad & |K_cG(S)| = 0.425 \times 10^{-4} [\text{rad}/(\text{ft}/\text{sec}^2)] \\ & \phi_\theta = 121^\circ \end{aligned} \quad (16)$$

The following filter was designed to meet these requirements:

$$K_cG(S) = 10^{-3} \left[ \frac{S^2 + 2(0.07)10S + 10^2}{S^2 + 2(0.7)10S + 10^2} \right] \times \left[ \frac{S^2 + 2(0.2)30S + 30^2}{S^2 + 2(0.2)5S + 5^2} \right] \left( \frac{5}{30} \right)^2 \left[ \frac{60^2}{S^2 + 2(0.5)60S + 60^2} \right] \quad (17)$$

A frequency response of this filter is shown in Fig. 4. The 60 rad/sec lag was used to represent the power cylinder dynamics. No special effort was made to define the simplest and most tolerant filter since its purpose was solely to demonstrate the validity of the synthesis technique. Figure 5 is a frequency response of the vehicle-system combination. The following points should be noted about this diagram: 1)



**Fig. 6 Mean square acceleration at the pilot's station vs frequency for a unit gust input.  $M = 0.54$ ;  $h = 5,000$  ft;  $W = 585,000$  lb.**

since  $90^\circ \leq \phi_{\delta\delta} + \phi_g \leq 270^\circ$  at essentially all points between the first and fourth modes, this phase constraint has been satisfied; 2) the fourth mode is gain stable with a gain margin of 5 db; and 3) the system gain margin is 10 db.

Figures 6 and 7 show the PSD plots of the acceleration at the pilot's station and at the aft cabin with this control system at the Mach 0.54, 5000-ft flight condition. It is observed that the required attenuation has been met at the first and second modes. It falls short of the objective of 0.25 at the third mode since the filter provided only half the required gain as evidenced in Fig. 5. Nevertheless, the constraints of the problem have essentially been satisfied. The percent reduction in the rms acceleration as the pilot's station was 42% and at the aft cabin it was 41%. Thus, the rms acceleration levels have been favorably reduced while at the same time adequate stability margins have been maintained. This same system was also evaluated at the Mach 0.9, 30,000-ft flight condition. Figures 8 and 9 illustrate the PSD plots of the acceleration at the pilot's station and at the aft cabin. Essentially the same performance was obtained at this condition as was obtained at the Mach 0.54, 5000-ft flight condition.

It was found that with this system some coupling with the rigid body mode was evident. To avoid this coupling, the filter would have to be modified slightly to provide additional attenuation at the rigid body mode. Since the objective herein was to demonstrate the synthesis technique primarily over the frequency range of the structural modes, this coupling was ignored. Of course, in any real application, this coupling would be taken into account in the design of the filter.

### Conclusions

The example described in the paper illustrated that the synthesis technique is an effective tool for use in the design of an elastic mode suppression system. An understanding of a few simple formulas and a knowledge of frequency response analysis enables the control system designer to readily acquire a working knowledge of the technique. The synthesis procedure has the following advantages.

- 1) It provides insight into the nature of the problem through the use of PSD plots and vector diagrams. This feature is especially apparent when analyzing the inter-modal coupling.
- 2) It provides the capability to handle unsteady aerodynamics due to the reliance upon a frequency response format for the vehicle data.
- 3) Once a filter-sensor combination has been selected, the synthesis procedure can be worked in reverse to establish the resulting performance. This feature enables the designer to assess readily any compromises in system design.

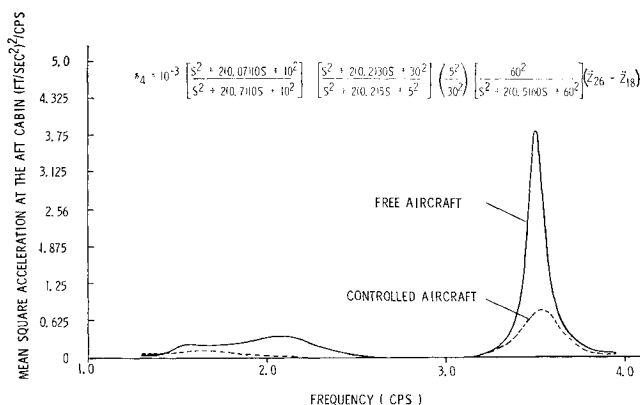


Fig. 7 Mean square acceleration at the aft cabin vs frequency for a unit gust input.  $M = 0.54$ ;  $h = 5,000$  ft;  $W = 585,000$  lb.

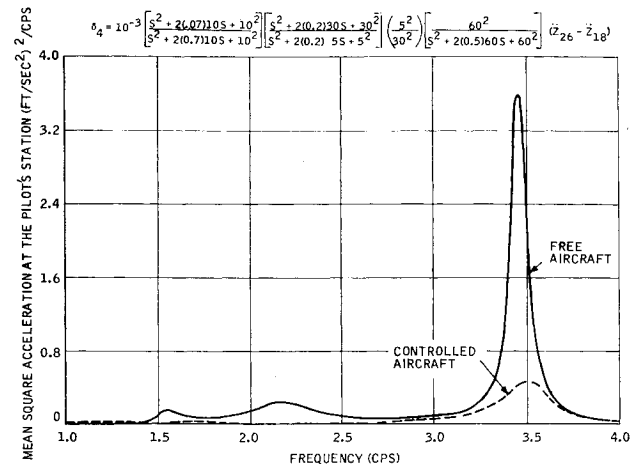


Fig. 8 Mean square acceleration at the pilot's station vs frequency for a unit gust input.  $M = 0.9$ ;  $h = 30,000$  ft;  $W = 585,000$  lb.

4) Given the vehicle data in frequency response form, all the computations can easily be worked by hand. The process, however, could be programmed on a computer should it be desirable to do so.

5) The synthesis technique is not limited by the order of the vehicle dynamics.

In considering the final point, it should be remembered that although the technique can be applied to a vehicle regardless of the number of structural modes, there are a few applications where the technique would be of limited use. For example, it is desirable to separate the rigid-body control problem from the structural mode control problem. If this is not possible through the selection of the control forcing function, sensor configuration, and filtering, it may be difficult to provide simultaneously both rigid-body handling qualities and adequate structural mode suppression. In one respect, optimal control techniques may provide an advantage over the technique described herein. Use of optimal control techniques may offer an advantage in designing a system where several forcing functions are considered simultaneously. Clearly, the technique described would become rather cumbersome in this case.

It is anticipated that this synthesis technique would be applicable to a variety of flexible aircraft, but in particular to large transports such as the SST. Further development of this technique is expected to lead to application in other areas such as in reducing cyclic stress important in fatigue on vehicles like the Boeing 747 and the Lockheed C-5A.

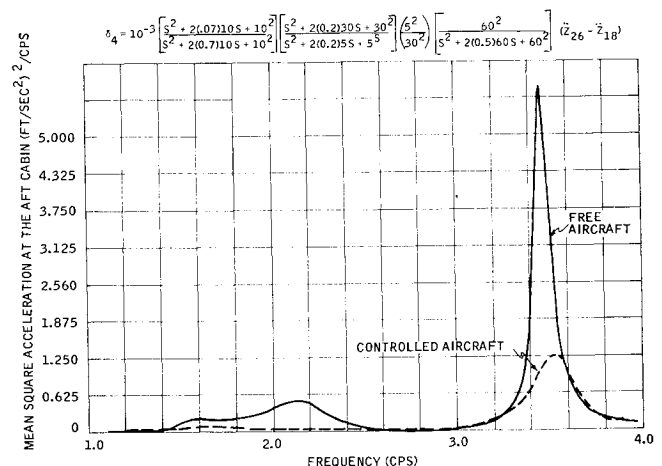


Fig. 9 Mean square acceleration at the aft cabin vs frequency for a unit gust input.  $M = 0.9$ ;  $h = 30,000$  ft;  $W = 585,000$  lb.



## Appendix A: Wind Model

The turbulence power spectral density function used in the analysis is given by

$$\Phi(\Omega) = \sigma_G^2 \frac{L}{\pi} \left[ \frac{1 + \frac{8}{3} (1.339L\Omega)^2}{[1 + (1.339L\Omega)^2]^{11/6}} \right] \frac{(\text{fps})^2}{\text{rad/ft}} \quad (\text{A1})$$

where  $\Omega = \omega/V$  = reduced frequency (rad/ft),  $V$  = aircraft velocity (fps),  $L$  = characteristic length (ft),  $\sigma_G$  = rms gust

velocity (fps), and

$$\int_0^\infty \phi(\Omega) d\Omega = \sigma_G^2$$

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- <sup>2</sup> Wykes, J. H. and Mori, A. S., "An Analysis of Flexible Aircraft Structural Mode Control, Part I," Tech. Documentary Rept. FDL-TDR-65, Aug. 1965, North American Aviation Inc., Los Angeles, Calif.

MARCH-APRIL 1968

J. AIRCRAFT

VOL. 5, NO. 2

# Automatic Flight Control System for Automatic Terrain-Following

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This paper discusses the design, simulation, and flight testing of an automatic flight control system (AFCS) developed under Project 666A† to provide an automatic terrain-following capability in a supersonic, fighter-bomber type of aircraft. Functional operation and features of the high performance, very reliable fixed-gain AFCS are described. Excellent terrain-following performance using AFCS hardware integrated with a flight simulator was achieved in a six-degree-of-freedom simulation program. Radar system failure effects on terrain-following performance and flight safety were studied and documented for Air Force review. Comparisons of flight test and simulation study results show very close correlation. The 666A AFCS will provide the high performance and high degree of reliability required to perform the automatic terrain-following task. Further flight test development and evaluation of the automatic terrain-following system will be performed by the Air Force at Wright-Patterson Air Force Base.

## I. Introduction

### Background and Goals of 666A Program

McDONNELL, General Electric, and Texas Instruments were selected by the U. S. Air Force to participate in the Project 666A Automatic Terrain-Following Program. This program was conducted at McDonnell from June 1965 through December 1966. The goals of Project 666A were to develop and demonstrate an automatic terrain-following capability in the vertical plane, and to provide precise lateral control for guidance and navigational course direction in the horizontal plane of a high-performance, fighter-bomber type of aircraft, such as the McDonnell F-4.

### Responsibilities of Associate Contractors

Texas Instruments Inc. was responsible for the design, development, and fabrication of the AN/APQ-101 Forward Looking Radar and the Terrain-Following Computer. General Electric was given responsibility for the design and fab-

rication of the AFCS hardware, performing analytical design studies, accomplishing flight-worthiness tests of the AFCS hardware, and for participating in the flight test evaluation of the AFCS.

McDonnell's primary responsibilities in the 666A program were 1) to participate in the design and development of the 666A Automatic Flight Control System (AFCS), 2) to install and integrate the AFCS and radar into the aircraft, and 3) to perform the flight test evaluation of the AFCS. This paper summarizes the results of analysis, simulation, and flight test accomplished in developing and evaluating the AFCS for use in providing the automatic terrain-following capability. Simulations and flight tests were conducted to determine that the performance of the AFCS met the specified design requirements.

## II. 666A Automatic Flight Control System (AFCS)

† The 666A AFCS was designed to provide the high degree of performance and reliability required for automatic terrain-following in the sensitive low-altitude, high-speed (LAHS) environment.

### Features of the 666A AFCS

‡ The 666A AFCS provides the following features in pitch, roll, and yaw channels.

Presented as Paper 67-569 at the AIAA Guidance, Control, and Flight Dynamics Conference, Huntsville, Ala. August 14-16, 1967; submitted July 31, 1967; revision received January 15, 1968.

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‡ Work funded by Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio, Contract AF33(657)-13543.